Roche Limit

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1 Introduction

In the planetary science book background information is provided on the Roche limit, the essence of the problem is to find the minimal distance between een planet and a satellite so that the tidal acceleration $\overline{a}_t$ and the binding acceleration $\overline{a}_b$ balance at the satellite. Thus:

$$\overline{a}_t = \overline{a}_b \tag{1}$$

In the following we will assume that:

- The planet has a mass M_p and a radius r_p, its gravitational constant is μ_p and the its density is ρ_p
- The satellite has a mass M_s and a radius r_s, its gravitational constant is μ_s and the its density is ρ_s
- The separation distance between planet and satellite is called d

So far we have not said where the balance holds and how the binding or tidal acceleration should be calculated. In fact, this depends on how you exactly define the problem. The straightforward method is to assume that the satellite is at distance d and that the balance holds at its surface. In this case you get, see also the planetary sciences book for more detail:

$$\frac{3\mu_p}{d^3} r_s = \frac{\mu_s}{r_s^2} \tag{2}$$

where the left hand side is obtained via a Tayler series approximation of the gravitational attraction at the satellite's center times the linearization distance r_s. The right hand side is the counterbalancing acceleration at the satellite's surface. We arrive at the expression:

$$d^3 = 3\frac{\mu_p}{\mu_s} r_s^3 \tag{3}$$

where the ratio of the gravitational constants of planet and satellite can be reduced to:

$$\frac{\mu_p}{\mu_s} = \frac{\rho_p}{\rho_s} \frac{r_p^3}{r_s^3} \tag{4}$$

so that the Roche limit becomes:

$$d = 1.44 \left(\frac{\rho_p}{\rho_m}\right)^{1/3} r_p \tag{5}$$

The fact 1.44 can be raised to a higher value for the reasons explained on pages 405–406 in the book. The following explains such a situation where we consider two satellites each with radius r_s stuck together (by gravitational forcing) so that they are separated at a distance $2r_s$. The balance between tidal forcing (and not net gravity forcing as in the book) and binding now becomes:

$$\frac{2\mu_p}{d^3} 2r_s = \frac{\mu_s}{(2r_s)^2} \quad \Rightarrow \quad d = 2.52 \left(\frac{\rho_p}{\rho_m}\right)^{1/3} r_p \tag{6}$$

and this answer is about right, that is, if you include oblateness and rotation for the satellite in the problem then the correct answer (d = 2.456 etc) is found. But even this situation is an assumption because real moons will resist destruction by tidal forcing because of their tensile strength. Examples of Moonlets that orbit within the Roche limit of a planet are Phobos in orbit around Mars, Metis, Adrastea and Almathea for Jupiter and Pan, Atlas, Prometheus and Pandora for Saturn, Cordelia, Ophelia, Bianca and Cressida for Uranus and Naiad, Thalassa and Despina for Neptune. Over time these moonlets will disappear because the most likely scenario is that they lose altitude so that the tidal forcing will increase.

Other examples of objects that are destructed due to tidal forcing are comets. Shoemaker Levy 9 approached Jupiter within the Roche limit and several other comets have been torn apart near the Sun.